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Groupe Consultatif

DIVERSIFICATION

Technical paper

31 October 2005

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1. Preface

The intention of this paper is to present a method that can be used to calculate the diversification effects for economic capital. Theoretically accepted methods include the use of the combination of the risk measure Tailvar (or CTE or Expected Shortfall) and Copula-functions. Both methods will only work "perfect" in case good information about the tail is available. In practice this will be difficult. In most cases only information is available regarding distributions and dependencies between risks under "normal" circumstances. The further in the tail we need this information the harder it will be to get it. Therefore we have adopted a practical method that will give acceptable results, not too far from the "theoretically correct" outcomes and more easy to understand.

The following technical issues arising in the quantification of diversification effects at Business unit level and group level are described in this paper:

1. Risk Types
2. Levels of diversification
3. Copulas versus Tail correlation matrix
4. Estimation of Tail correlation factors
5. Testing the correlation factors

This paper is based on a bottom up approach. In order to get the total capital needed at the highest level of a group we start with the calculation at the lowest level the sub risks. The issue is then how to combine these sub risks to obtain the capital at various levels of an organisation.

Alternatively a top down approach can be applied i.e. aggregating the exposures throughout the group for each risk and then assessing the required capital using scenario analysis to identify the key risk drivers at group level and modelling these. These methods can be difficult to set up for complex conglomerates (consistency in scenarios, difficulty setting appropriate scenarios), also we want to have not only the total capital but also the capital at intermediate organisational levels, so we need an allocation system.

Conclusions of this paper

- Bottom up approach should always start at the lowest level of risk classification.
- Tail-correlation is a good and acceptable alternative for the use of the complex Copula-method, particularly in case we only need the correlation at one point of the distributions.
- Estimation of tail-correlation factors can only be done using "expert – opinion", starting with experience analysis on dependencies between risks in "normal" cases. Sensitivity testing of the most important factors is needed to know where "most energy" should be put in.
- The diversification models can always be improved.

In the appendix B an example shows a total numeric overview of all the issues described in this paper.

Appendix A gives information on the theory behind copulas.

Appendix C presents thinking on the wider issues arising when considering the solvency assessment of an entity that is part of a financial group. A preliminary version was shared with CEIOPS in June 2005 and a few adjustments have subsequently been made.

2. Risk Types

The set up of the risk types is comparable with the method used in the IAA approach (see Chapter 5 of “A global Framework for Insurer Solvency Assessment” from the IAA Insurer Solvency Assessment Working Party) such that a distribution around an expected value of a risk can be based on 3 parts:

- Volatility

Volatility is the risk of random fluctuations in either the frequency or severity of a contingent event. This risk is “diversifiable”, meaning that the volatility of the average claim amount declines as the block of independent insured risks increases.

- Uncertainty (parameter/model)

Uncertainty is the risk that the models used to estimate the claims or other relevant processes are misspecified or that the parameters within the model are misestimated. Uncertainty risks are non-diversifiable. Increasing portfolio size will not reduce relatively the risk.

If there is a larger volume of relevant data uncertainty in parameter selection is reduced. However for a given level of such uncertainty writing more volume does not reduce parameter uncertainty.

- Extreme events

Extreme events are events with high-impacts and low-frequency. They will cause fluctuations greater than normally arise from normal modelled fluctuations. In most cases there are not sufficient observations available to quantify these risks solely from past experience. Also changing conditions mean that the past is not necessarily a guide for the future. In addition we note that risks that are normally almost independent can be more strongly correlated when extreme events occur.

Particularly the underwriting risks are split into the three earlier mentioned sub risks: volatility – uncertainty – extreme event risk. For example mortality risk can be split into the following sub risks;

- volatility
- uncertainty trend (uncertainty in the estimation future mortality)
- uncertainty level (uncertainty in the level of mortality for insured population)
- Calamity (extreme event risk for mortality, example Spanish flu – natural catastrophes like earthquakes, causing high numbers of deaths etc.)

3. Levels of diversification

Because of:

- law of large numbers
- opposite risks
- unconnected risk
- risks that are less than 100% interdependent

the combining of the several distributions of all the sub risk types will cause a reduction of the total risk. This diversification is critical to risk management. Diversification forms the foundation of insurance and is the key-stone on which important risk management processes rest.

This combining of risks that are not totally dependent causes the diversification effect: the total capital related to the combination of sub risks will be equal or lower than the sum of the capitals for each sub risk.

Part of the mentioned diversification effects, like the law of large numbers, will already be included in the models used to calculate the capital, e.g. the volatility over the modelled group of business. Also in case opposite risks exist within the modelled group of business this effect will be reflected in the modelling of the capital model. This latter one is also called netting effect. The diversification between the risk types and because of combining the modelled blocks of business is done in the diversification model.

The diversification effect can be calculated at several levels:

a. Between sub-risks, within a risk type

In this level sub risk types are combined into one risk type. This can be done for presentation reasons. Example: sub risks like mortality trend uncertainty, level uncertainty, volatility and calamity are combined into one risk type: Mortality.

b. Between risk-types within a business unit (BU)

The result is the capital for the stand-alone business unit. It contains the diversification between the risk types and perhaps in combining blocks of business leading to further volatility diversification (law of large numbers)

c. Between BU's within the group or part of the group.

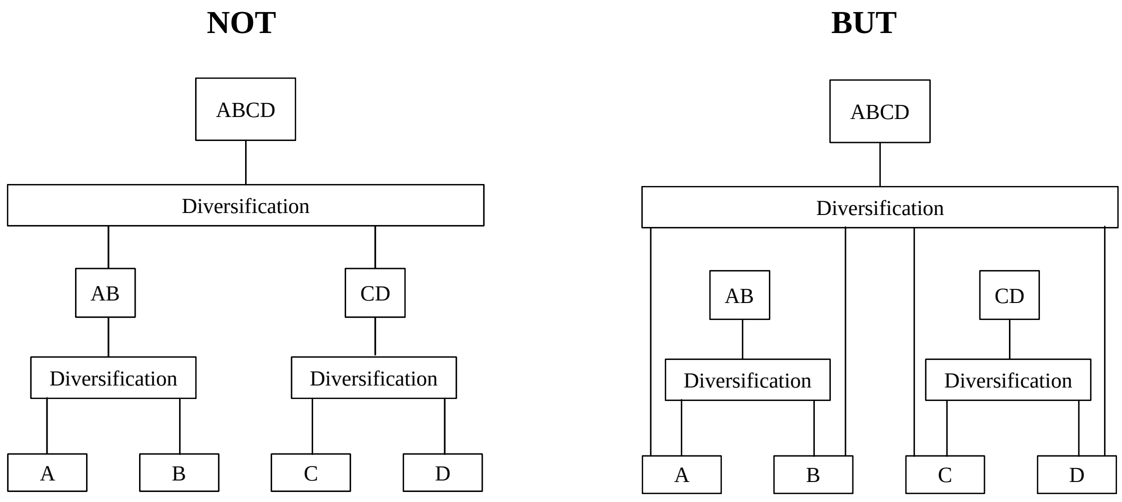
Combining several BU's into one group will result in further diversification because of adding volatility parts, combining risk-types over a larger range than in level 2 and also within a risk-type, in most cases depending on geographic and economic situations.

Of course diversification can also be calculated at other levels, in between the levels mentioned above, for example between entities within one country or at an extreme between the total of sub risks compared to the group total.

As mentioned at the start of this memo we describe here the bottom up approach. Starting with the capitals for each sub risk within an entity we want to derive the capitals needed at the higher levels, like entity-level or group-level.

It is important to notice that the bottom up calculation ALWAYS has to start at the lowest level

So, the Bottom up approach is



The following explains why it is necessary to work from the lowest level.

A risk like "mortality" contains several risk drivers, as described above (uncertainty level- uncertainty trend- volatility- calamity). Therefore the total correlation will depend on the weight between these risk drivers. This weight will differ by type of product, type of business and so on. An example:

Suppose we have the following risks and required capital for each risk on a stand alone basis:

Table 1

Risk	Capital
A	1000
B	200
C	2000
D	500

And the following correlation factors between those risks:

Table 2

	A	B	C	D
A	1			
B	0.50	1		
C	0.75	0.75	1	
D	0.50	0.50	0.25	1

Using these correlation factors the following total capitals can be calculated:

Table 3

Risk	Capital
A+B	1114

C+D	2179
A+B+C+D	3192

From this we can derive that the correlation factor between (A+B) and (C+D) is:
0.865 $(= \frac{3192^2 - 1114^2 - 2179^2}{2 \times 1114 \times 2179})$ to get the total capital for A+B+C+D: 3192.

Suppose the correlation factors stay the same but during a year the capitals change from the ones in table 1 into (this could arise from changes in business volume or of product mix):

Table 4

Risk	Capital
A	1100
B	300
C	1800
D	800

Using the same correlation factors (table 2) the following new total capitals can be calculated:

Table 5

Risk	Capital
A+B	1277
C+D	2145
A+B+C+D	3336

From these capitals we can derive that while the dependencies between the risks did not changed at all the correlation factor between (A+B) and (C+D) changed from: **0.865** into **0.895** to get the total capital for A+B+C+D: 3336.

Conclusion:
Only correlation factors set at the lowest level are unequivocal and stable over time. Thus capitals at higher levels should always be derived starting at a lower level. Deriving a higher level aggregation from a lower level aggregation above the sub risk level risks will result in misleading outcomes at the entity or group level.

The fundamental analysis starts with looking at risk drivers or events that cause changes in the observed events. The correlation between sub risk categories will depend on the consequences arising from these risk drivers when applied to the portfolio of business. While many direct writers portfolios will be relatively stable year on year it is good practice to review the validity of the sub risk correlations. Consideration needs to be given to changes in characteristics or mix of products that may impact dependencies.
An example: in most cases low interest rates results in a loss, but for some products that is not the case as increasing interest rates causes losses. Sometimes this interest rate risk exists on both sides. Therefore a change in product mix can change the sign of the risk that impacts the correlations related to this risk.

4. Copulas versus the use of Tail correlation factors

The IAA proposes to use Copulas as the theoretically correct method to calculate diversification effects. Indeed the use of a “standard” correlation matrix is wrong. Copulas have the advantage that they can be used to accurately combine other distributions than from the “Normal Family” and that they can recognise dependencies that change in the tail of the distribution.

Severe incidents can impact risks that are normally independent. Example: normally market risk and mortality risk will be independent. But when a severe pandemic like the Spanish Flu would happen with world-wide millions of deaths this will certainly have economic consequences and will also impact market risk (for example equity-risk).

In practice combining several distributions implies that the dependency in the tail is higher than on average.

A problem with the use of Copulas is that it is very complex in the case that a rather large number of distributions have to be combined. Also there is generally limited data available to estimate the copula function in the tail. Given these observations many practitioners consider that a simpler approach can deliver acceptable results.
A more detailed explanation of Copulas can be found in Appendix A

An alternative for Copulas can be the use of an adjusted correlation matrix.
The result of Copulas will be a combined distribution function. However, we are only interested in the part of it around the confidence level. Instead of filling a matrix with correlation factors that describe the average dependency across the whole distribution, we estimate only the dependency at the point we want to know it. The use of an adjusted correlation matrix filled with tail-correlations will only get reliable results in a small part of the distribution. We are only interested in the tail (above the confidence level), so the use of a tail correlation matrix is a good alternative. Be aware that tail correlation factors are not used for standard deviations but are applied on capitals.

Simulation models illustrate the use of tail correlation factors.

Simulation 1: Two independent normal distributions:
Table 6

Correlation factor : 0			
	Conf. 90%	Conf. 99%	Conf. 99.95%
Risk 1	1.3	2.4	3.2
Risk 2	1.3	2.3	3.2
Combined exact	1.8	3.2	4.5
Using Cor. Factor.	1.8	3.2	4.5

As expected the use of the correlation matrix in this case produces the correct result.

Simulation 2: Independent risks - risk 1: log-normal; risk 2: Poisson (10)
Both types of distributions are common in economic capital calculations.

Table 7

Correlation factor : 0			
	Conf. 90%	Conf. 99%	Conf. 99.95%
Risk 1	1.8	8.8	24.1
Risk 2	4.0	8.0	12.0
Combined exact	4.7	11.9	24.4
Using Cor. Factor.	4.4	10.8	27.0

The method using the correlation matrix is less accurate, although the “mistake” is not extreme. But we need to be careful using the correlation matrix method, even in case of independent risks.

Simulation 3: Two normal distributions, but with a high dependency. The dependency is formed by: the result of distribution 1 gives the expected value of distribution 2.

Table 8

Correlation factor : 0.70			
	Conf. 90%	Conf. 99%	Conf. 99.95%
Risk 1	1.2	2.2	3.3
Risk 2	1.8	3.3	4.7
Combined exact	2.8	5.1	7.4
Using Cor. Factor.	2.8	5.1	7.4

As expected the method with correlation factors works correct.

Simulation 4: like 3 but dependency only in tail of distributions (above 2 sigma’s)

Table 9

Correlation factor : 0.12			
	Conf. 90%	Conf. 99%	Conf. 99.95%
Risk 1	1.2	2.3	3.3
Risk 2	1.3	2.7	4.4
Combined exact	1.8	4.8	7.3
Using Cor. Factor.	1.9	3.8	5.9

The method with correlation factors produces incorrect outcomes, particularly in the far tail. The Var’s are underestimated at the high confidence levels.

Simulation 4b: like 4 but with adjusted correlation factors

Table 10

Correlation factor : 0.12, adjusted for tail correlation : 0.70			
	Conf. 90%	Conf. 99%	Conf. 99.95%
Risk 1	1.2	2.3	3.3
Risk 2	1.3	2.7	4.4
Combined exact	1.8	4.8	7.3
Using Cor. Factor.	2.3	4.7	7.2

The adjusted tail correlation is based on the fact that the same kind of correlation is used as in scenario 3. The table shows much better outcomes using adjusted correlation factors. The deviation with the correct capital is small precisely at the

higher confidence levels. On the other hand: the same adjusted correlation factor can not be used at the lower confidence levels. That is why tail correlations factors can not be used to produce a complete combined distribution. The method is only applicable in a narrow range of confidence levels. Therefore it will give acceptable outcomes for VAR, but will be less convincing for TailVar, where we need to describe the dependency over the whole tail of the distributions.
In the example in 4b an extreme tail dependency is presented. That results in an adjustment from 0.12 to 0.70. In practice this tail dependency will be more smoothed and usually results in lower adjustments.

5. Estimation of tail-correlation factors

The estimation of the correlation between two risks under extreme circumstances is subject to the same uncertainties as the selecting of copula functions. There will never be enough data for a reliable estimation. By definition extreme situations will not happen frequently. Extreme events that will happen in the future did not happen yet in the past. The only possibility we have is the use of scientific evidence on dependencies, based on semi-worse case events in the past and expert opinion and to get an agreement between industry partners and the regulators.

As a result outcomes are by nature not exact and we should not aim to be more precise. The expert opinion should result in words that can be translated into numbers.

Table 11

Independent	0
Some correlation	0.25
Significant correlation	0.50
High correlation	0.75
Full correlation	1

With a sensitivity test it is possible to find the (for example) ten most important correlation factors. Those ten factors can cause a significant part of the total diversification effect. It is important to put more energy in assessing these factors and less time on the others. In case of high sensitivity smaller steps than 0.25 could be considered.

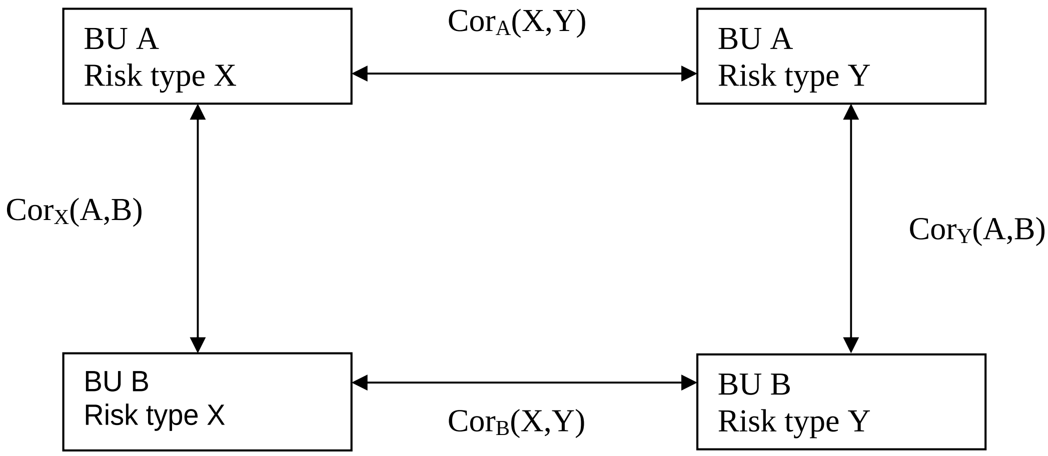
Three levels of correlation factors need to be estimated:

1. Between (sub)risks within an entity
2. Between entities within a (sub)risk
3. Between several risk types and several entities

With the expert opinion level 1 and level 2 can be set. Level 2 will often depend on geographic and/or economic situations.
Level 3 can be derived from 1 and 2, i.e. there is no need to estimate separately the level 3 correlation factors. An approach is described below.

Also correlation factors are needed between sub risk X of BU A and sub risk Y of BU B. These can be derived from the others.

Approximation of correlation between risk type X in BU A and risk type Y in BU B.



Correlation factor: $\dfrac{Cor_X(A,B)+Cor_Y(A,B)}{2}\times\dfrac{Cor_A(X,Y)+Cor_B(X,Y)}{2}$

.

This factor is the product of the averages of the two sets of factors involved. In this way the results are logical.

6 Setting and testing the correlation factors

6.1 Test of the impact on the diversification effect of the several correlation factors.

It is important to know the correlation factors between the risk-types with the highest impact. This can be done by setting the correlation factors one-by-one at 1. So the impact will be high when capitals involved are high in combination with a rather low correlation factor.
This analysis can be made by setting the correlation factors for the risk between BU's all at 1. So no diversification between the BU's for this risk is allowed.
This type of correlation only impacts the diversification level 3 (between BU's).

6.2 Sensitivity test of correlation factors

The correlation factors are generally set up in steps of 0.25. It is possible that this step is too high for some risk combinations. In case the impact of this step is high it should be examined if the step should be smaller. This impact is calculated using a step of 0.25 downwards. An upwards step will not be exactly the same but will be close enough for conclusions. Problem with upwards steps are the factors that are already at 1.

7 Allocation diversification effects

Once the diversification effects, for example at group level, are calculated we want to know what amount the group can allocate to each of the entities. So we want to know how to split the diversification to each of the entities. This can be done in two ways:
1. Give each entity the same percentage reduction of the capital (proportional)
2. Take into account the contribution of each of the entities to the total diversification (marginal).

From a technical point of view the marginal way to allocate back the diversification effects is more logical, particularly in analysing risk types.

Calculating proportional allocation:
With C_g=total diversified capital group and C_i=stand alone capital for entity i the diversified capital for entity i C_i^d follows:

$$C_i^d = C_i \times \frac{C_g}{\sum_j C_j}$$

In this way each of the entities gets the same ratio as reduction because of group diversification, independent of the contribution of that risk to the total diversification.

Calculating marginal allocation:
We wish to calculate how the total capital (group capital) is affected by the inclusion of each entity. This is done by calculating how much the total group capital increases for a small increase in risk i or entity i, by taking the partial derivative of the capital for the portfolio with respect to the capital for entity (or risk) i and multiplying by the stand alone capital for entity (or risk) i (C_i).

So:

$$C_i^d = C_i \times \frac{\partial C_g}{\partial C_i} =$$
$$= C_i \times \frac{\sum_j C_j \rho_{ij}}{C_g}$$

It can be proved that the sum of all the diversified capitals for each of the entities equals the group capital:

$$\sum_i C_i^d = \sum_i C_i \times \frac{\sum_j C_j \rho_{ij}}{C_g} =$$

$$\frac{\sum_i \sum_j C_i C_j \rho_{ij}}{C_g} =$$

$$= \frac{C_g^2}{C_g} = C_g$$

In the example you can find in Appendix B the working of the 2 methods is shown.

In practice for technical analyses the marginal method is preferable.

Appendix A

Using copulas in the measurement of the diversification effect

1- Copulas Theory: Some basic concepts/results.

Suppose that the random vector $X = (X_1,...,X_d)'$ defines the risk that a conglomerate group or entity faces in each of its *d* sub-risks, assuming a bottom-up approach.

In insurance and banking it is absolutely necessary to have a statistical analysis of the risks that allows the study of their inter-dependence, as this has practical consequences in risk management.

Without loss of generality, we can assume the behaviour of the conglomerate is described by a joint distribution *F* with continuous marginal distributions $F_1,...,F_d$ that describe the behaviour of each one of the *d* sub-risks.

So, if we want to know what is the probability that each one of the risks, for example, assumes a value below a certain level $x_1,...,x_d$, we may write

$F(x_1,...,x_d) = P(F_1(X_1) \le F_1(x_1),...,F_d(X_d) \le F_d(x_d))$ in which it is known that for $i=1,\dots,d$ we have that $F_i(X_i)$ is a uniform random variable, in the interval (0,1).

In this case we feel the need to know *F*. This can be done by the definition of each individual distribution (marginal distributions) and coupling them through the definition of a Copula function,

$$F(x_1,...,x_d) = C(F_1(X_1),...,F_d(X_d))$$

The Sklar Theorem, demonstrates that, if *F* is a *d*-dimensional distribution with univariate margins, $F_1,...,F_d$ then there is a unique copula such that

$$F(x_1,...,x_d) = C(F_1(X_1),...,F_d(X_d))$$

Conversely, if C is a d-dimensional copula, and given $F_1,...,F_d$ as the univariate continuous distribution functions, then $C(F_1(X_1),...,F_d(X_d))$ is a joint *d*-dimensional distribution function with univariate margins $F_1,...,F_d$.

A Copula is a function that links the distribution function of different random variables within a stochastic dependence context.

Modelling marginal distributions together with copulas provides a mode for the aggregate portfolio accounting for dependence between lines of business.

Appendix A

Copulas have many different forms, for example :

- For the bivariate, logist also known as Gumbel, distribution function, the copula is

$$C_\theta^{Gu}(u_1,u_2)=\exp\left\{-\left[\left(-\log u_1\right)^\theta+\left(-\log u_2\right)^\theta\right]^{\frac{1}{\theta}}\right\}$$

  $\theta \geq 1$ is the parameter that controls the level of dependence between the sub risks X_1 and X_2. For $\theta=1$ we have independence.
- For the d-dimensional Gaussian distribution function, for a mean zero vector and correlation matrix Σ, the copula is

$$C_\Sigma^{Ga}(u_1,...,u_d)=\Phi_d\left(\Phi_1^{-1}(u_1),\Phi_1^{-1}(u_2),...,\Phi_1^{-1}(u_d)\right)$$

- The copula combining independent distributions is $\mathrm{C}(\mathrm{u}_1,...,\mathrm{u}_\mathrm{d})=\prod_{\mathrm{i}=1}^{\mathrm{d}}\mathrm{u}_\mathrm{i}$

In practical terms, a copula is the joint probability that some risk X_1 lies below its u_1 quantile $\left(F_1^{\leftarrow}(u_1)\right)$ and X_2 lies below its u_2, and so on.

2- Measures of dependence.

It is important to use copulas to get a better understanding (and so a measure) of the kind of dependence that exists, especially in the tails (because its with the case of extreme outcomes that we must worry!) of the joint distribution function, which can be done through the tail dependence coefficient (λ). This method is preferable to using only the simple linear correlation, which plays a central role in financial theory (as can be seen in the CAPM), but which is only theoretically correct with elliptical distributions (distributions whose density is constant on ellipsoids), such as the Normal.

2.1-Simple Linear Correlation.

The simple linear correlation $\rho\left(X_1,X_2\right)$ between two random variables X_1 and X_2 is defined by:

$$\rho\left(X_1,X_2\right)=\frac{\operatorname{cov}\left(X_1,X_2\right)}{\sqrt{\operatorname{var}\left(X_1\right)\operatorname{var}\left(X_2\right)}}\in\left[-1;1\right]$$

where $\operatorname{cov}\left(X_1,X_2\right)=E\left(X_1X_2\right)-E\left(X_1\right)E\left(X_2\right)$. When working with a vector X the var-cor matrix corresponds to: $E\left[\left(X-E\left(X\right)\right)\left(X-E\left(X\right)\right)'\right]$.

The linear correlation has the following proprieties:

- If X_1 and X_2 are independent, then $\rho\left(X_1,X_2\right)=0$
- In general, if $\rho\left(X_1,X_2\right)=0$ it doesn't mean that X_1 and X_2 are independent.
- If $\left|\rho\left(X_1,X_2\right)\right|=1$, X_1 and X_2 are perfectly linear dependent, meaning that

$X_2 = \alpha + \beta X_1$, with $\alpha \in R$ and $\beta \neq 0$, in which if $\beta > 0$ it is a perfectly positive linear dependence, and in the other case it is a perfectly negative linear dependence.

- Is invariant under strictly increasing linear transformation, $\rho(X_1, X_2) = \rho(\alpha_1 + \beta_1 X_1, \alpha_2 + \beta_2 X_2)$, but it isn't for non-linear strictly increasing transformation.

A correlation is only defined when the variances of X_1 and X_2 are finite.

This restriction to finite variance models is not ideal for a dependence measure and can cause problems when we work with heavy tailed distributions. So actuaries who model losses in different business lines with infinite variance distributions may not describe the dependence of their risks using correlations.

Besides, linear correlations don't tell us anything about the degree of dependence in the tail of the underlying distribution.

Only in the case of assuming distributions belonging to the "Normal family", can we say that the marginal distributions and pairwise correlations determine the joint distributions of a vector of risks. With these conditions, it is natural to use:

- the correlation matrix as a summary of the dependence structure of constituent risk.
- the $VaR_\alpha(X) = \inf\{x : F_X(x) \geq \alpha\}$ as a measure of risk, because it can satisfy the sub-additivity property: $VaR_\alpha(P_1 + P_2) \leq VaR_\alpha(P_1) + VaR_\alpha(P_2)$, with $\alpha > 0{,}5$ and P_1, P_2 portfolios obtained from a linear combination of risks with elliptical distributions.

A solution to measure the tail dependence, when using copula distribution with continuous marginal distributions is the tail dependence coefficient which is an asymptotic measure of dependence, specially focused on bivariate extreme values.

Let (U_1, U_2) be a vector of 2 random variables uniformly distributed on U(0;1), the tail dependence coefficient, (λ), exists since it can be obtained by:

- $\lim_{u \to 0^+} P(U_1 \leq u \mid U_2 \leq u) = \lambda_L$. So
  - If $\lambda_L \in]0;1]$ then C has lower tail dependence.
  - If $\lambda_L = 0$, then C does not have lower tail dependence.
- $\lim_{u \to 1^-} P(U_1 > u \mid U_2 > u) = \lambda_U$. So
  - If $\lambda_U \in]0;1]$ then C has upper tail dependence.
  - If $\lambda_U = 0$, then C does not have upper tail dependence.

In terms of quantiles, if X_1 and X_2 have continuous distribution functions, F_1 and F_2 respectively, then:

- $\lim_{u\to 0^+} P\left(X_2 \le F_2^{-1}(u) \middle| X_1 \le F_1^{-1}(u)\right) = \lambda_L = \lim_{u\to 0^+} \frac{C(u,u)}{u}$
- $\lim_{u\to 1^-} P\left(X_2 > F_2^{-1}(u) \middle| X_1 > F_1^{-1}(u)\right) = \lambda_U = \lim_{u\to 0^+} \frac{2u-1+C(1-u,1-u)}{u}$

It is possible to establish limits for copulas, known as the Fréchet bounds, which can be helpful in the interpretation of dependence. For every copula $C(u_1,...,u_d)$ the bounds are,

$$\max\left\{\sum_{i=1}^{d} u_i + 1 - d; 0\right\} \le C(u_1,...,u_d) \le \min(u_1,...,u_d)$$

The lower bound corresponds to the countercomonotonic copula in which X_2 is, strictly, decreasing function of X_1.
The upper bound is the comonotonic copula, representing the perfectly positive dependence.

Fréchet bounds for a multivariate distribution function F with margins $F_1,...,F_d$ can be obtained through

$$\max\left\{\sum_{i=1}^{d} F_i(x_i) + 1 - d; 0\right\} \le C(u_1,...,u_d) \le \min\left\{F_1(x_1),...,F_d(x_d)\right\}$$

3- Some remarks about diversification.

We have defined some concepts and results about copulas with the aim of incorporating them in the measurement of diversification for portfolios of non-normal risk. To do so, it is convenient to measure the gain of the diversification risk, and show some remarks about diversification. But before, we define two alternative risk measures.

One risk measure is $VaR_\alpha(X)$, the maximum possible loss, which is not exceeded with probability α ($\alpha = 95\% or 99\%$). Another risk measure is the expected shortfall $ES_\alpha(X) = E\left(X \middle| X > VaR_\alpha(X)\right)$. This ES is the conditional expected loss, given that the loss exceeds its VaR or the average of the $100\times\alpha\%$ worst cases, assuming the loss as a positive value.

$$ES_\alpha = VaR_\alpha(X) + \frac{1}{1-\alpha}\overline{F}_X\left(VaR_\alpha(X)\right)E\left(X - VaR_\alpha(X) \middle| X > VaR_\alpha(X)\right)$$

So, we can define the measure of diversification effects for economic capital as:

Appendix A

$$G^D = \sum_{i=1}^{d} VaR_\alpha\left(X_i\right) - VaR_\alpha\left(\sum_{i=1}^{d} X_i\right)$$

which is a positive value in case of the existence of a gain in using diversification.

Some insurance business unit (b.u.) has *n* policies, all of each with a insured sum equal to 1 (capital that must be payed in case of a total loss).

The company defines a probability of *p* as the probability of an only loss in a year, of any of the *n* policies. So, formally, the behavior of the loss of the *i* policies is $X_i \sim Bernoulli(p)$. The analysis of the business unit reveals that the behavior of the total loss $S = \sum_{i=1}^{d} X_i$, when the policies are independent, is $Binomial\left(n, p\right)$. Assume:

- $F_{S_{n_j}}(s)$ as the distribution function of the total loss with n_j policies;
- α as the security level
- $k > 0$ the increase in the dimension of the sample of policies, i.e. $n_{j+1} = k \times n_j$, as $j \to \infty$.

So, according to the dimension of the sample of policies n_j and the security level, the business needs to define different VaR, where $VaR_\alpha\left(S_{n_1}\right) = F_S^{-1}(s)$.

Following that, and according to the law of large numbers, for a particular j, assuming $j = 1$, it happens that:

$$\frac{VaR_\alpha\left(S_{n_2}\right)}{VaR_\alpha\left(S_{n_1}\right)} < k$$

So, when the number of policies increase by a factor *k* the VaR increases by a value much lower than *k*. This happens because large portfolios are less volatile than small ones.

But when it refers to very large portfolios, where $n_j \to \infty$, it happens that:

$$\frac{VaR_\alpha\left(S_{n_{j+1}}\right)}{VaR_\alpha\left(S_{n_j}\right)} \to k$$

Therefore for very large portfolios the extra diversification to be gained from further increase in size reduces and in extreme there is no further gain.

Appendix B

Example diversification and groups effects

In a simplified way all the issues described in this paper are presented. To keep it simple not all the possible risk are included, but only a limited number. Remember it is a simplified model, just to show how the models work.

The group contains 3 business units, spread over 2 countries. In table 1 you find an overview of the stand alone capitals involved.
Table1

		Country 1	**Country 2**	**Country 1**
	Risk type	BU 1 (life business)	BU 2 (annuity business)	BU 3 (P&C)
Life	Trend uncertainty	400	700	0
Life	Level uncertainty	300	600	0
Life	Volatility	150	10	0
Life	Calamity	100	0	0
Non life	Non cat uncertainty	0	0	200
Non life	Non cat volatility	0	0	20
Non life	Catastrophe risk	0	0	250
Market	Interest	1000	2000	300

In this example capital after diversification at BU and group level are analysed:

a. setting correlation factors
b. calculating diversified capital
c. allocation of diversification effects to lower levels
d. testing the impact of correlation assumptions
e. how to deal with diversification at group level

a. setting correlation factors

Because in this example we have 3 BU's and 8 different risk types the total correlation matrix with be 24x24, with 276 risk combinations to define. We do that in 3 steps.

First we define the correlation factors between the risk types, then the correlation factors within a risk type between the BU's and then the factors between different risk types and different BU's.
We take into account that we need adjusted factors for tail dependencies and for non-normality. The factors are as described in chapter 5 set in steps of 0.25.

In table 2 the correlation matrix is presented.

Table 2

		life				non-life			
		trend	**level**	**volatility**	**calamity**	**non cat unc**	**non cat vo**	**cat. risk**	**Interest**
life	**trend**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	level	**0**	**1**	**0.25**	**0**	**0**	**0**	**0**	**0**
	volatility	**0**	**0.25**	**1**	**0.25**	**0**	**0**	**0**	**0**
	calamity	**0**	**0**	**0.25**	**1**	**0**	**0**	**0.5**	**0.25**
non-life	**non cat unc.**	**0**	**0**	**0**	**0**	**1**	**0.25**	**0**	**0**
	non cat vol	**0**	**0**	**0**	**0**	**0.25**	**1**	**0**	**0**
	cat. risk	**0**	**0**	**0**	**0.5**	**0**	**0**	**1**	**0.25**
Market	**Interest rate**	**0**	**0**	**0**	**0.25**	**0**	**0**	**0.25**	**1**

Some explanation:
Between volatility and level uncertainty some (0.25) positive correlation is assumed. One cause of level uncertainty is observed volatility in the past.
In mortality volatility there is some dependency between the several risks. This dependency exists because of more or less deaths on result of climate or severe accidents. Calamity and non life catastrophe risk is set at "significant", because the calamity can be caused by a natural catastrophe.
Some positive correlation is assumed between interest rate risk and calamity/Catastrophe risk. A pandemic could cause millions of extra deaths world wide and would have some economic consequences. The same is valid for extreme catastrophes.

Now we define the correlation factors between the BU's within a risk type. In setting these we take into account if two BU's are within one region (or country) or not.

This leads to the following set of factors we use between the BU's.:

Table 3

		adjusted for tail dependencies and non normallity		
		Country C1-C1	Country C1-C2	
life	trend	1	0.75	In case of opposite sign: 0
	level	0.25	0	
	volatility	0.25	0	
	calamity	1	0.5	
non-life	non cat unc.	0.25	0	
	non cat vol	0.25	0	
	cat. risk	1	0.5	
Market	Interest rate	1	0.75	

Explanation:
Trend uncertainty within a country is set at one. This because the trend used in setting the Best Estimate mortality rates will be based on country population data. Still between countries not too far from each other the correlation will be high.
In our example we talk about life insurance (age group 25-65) and annuities (age group >60). Experience analyses in the Netherlands over the last century showed that the development over these age groups were not always the same, but even sometimes opposite. In case the sign of the risk is opposite (like in our example) the correlation factor is set at 0.

For uncertainty and volatility (both for life and non life) extra volatility is assumed because of climatologic impacts (like strong winters, hot summers). This causes some dependency within a country of countries nearby.

For calamity risk within a country the factor is set at 1. It is a severe event that will hit all the BU's in a country. Between countries it will depend on how far these countries are

between each other. Still there will always be a significant positive factor because of calamity as a result of a world wide pandemic.
For catastrophe within one country the factor should be 1. The extreme event will hit all the BU's within a country (also depending on the products). In countries nearby there will be some dependency, between countries far from each other the factor can be set at 0 (independent).
For interest rate risk the factor will depend on economic situations. The factors will be 1 within one monetary unit.

The factors between several risk types and several BU's are calculated using the formula in chapter 5 (page 11).

b. Calculating diversified capital.

With the correlation factor and the stand alone capital the diversified capital can be calculated at each level you want:

In total we have 24 stand alone capitals: for each of the 3 BU's 8 sub risks
C_i with i = 1 to 24
With the correlation factors between risk i and j: ρ_{ij}
The capital taken into account the diversification effect follows:

$$C_{div} = \sqrt{\sum_i \sum_j C_i C_j \sigma_{ij}}$$

It is also easy to use matrix algebra to make this calculation.
The result for our example group is:

Table 4
Example diversification insurance group

			Capital		Diversification within BU	Stand Alone BU		Diversification group	Total Group capital
Country 1	life 1	trend +	400						
		level	300						
		volatility	150						
		calamity	100						
		Interest	1000						
		Total	1950		782.74	1167.26			
				=	40%	60%			
Country 2	life 2	trend -	700						
		level	600						
		volatility	10						
		calamity	0						
		Interest	2000						
		Total	3310		1107.02	2202.98			
				=	33%	67%			
Country 1	non-life	non cat unc.	200						
		non cat vol	20						
		cat. risk	250						
		Interest	300						
		Total	770		287.92	482.08			
				=	37%	63%			
		Total group				3852.32		486.03	3366.29
							=	13%	87%

Note that trend+ and trend – are used to indicate that trend impacts liabilities in opposite directions for the mortality covers in BU 1 and the longevity covers in BU 2.

As you can see the reduction of the capital needed at BU level is 33% to 40%, depending on business and the spread of the capitals. Being part of the group results in another reduction of 13%. Again, this is only an example. In reality this number can be higher when more BU's are part of the group, or the risks involved have a lower correlation.

c. **Allocation of diversification effects to lower levels**.

There are two ways of allocating back diversification effects. This can be done in a proportional way: all a reduction of capital with the same percentage or using a marginal method: the allocation depends on the contribution of a risk or BU to the diversification effect. Also a combination is possible: within a BU marginal, from group to BU level proportional. In chapter 7 you can find the way how to calculate the methods.
In the next table the method are compared.

Table 5a

Example diversification insurance group

			Capital		Diversification within BU	Stand Alone BU		Diversification group	Total Group capital	Capital after alloc. prop	Capital after alloc. marg.
Country 1	life 1	trend +	400								47.53
		level	300								30.08
		volatility	150								11.14
		calamity	100								26.75
		Interest	1000								857.77
		Total	1950		782.74	1167.26				1019.99	973.27
				=	40%	60%					
Country 2	life 2	trend -	700								145.56
		level	600								107.39
		volatility	10								0.49
		calamity	0								0.00
		Interest	2000								1800.02
		Total	3310		1107.02	2202.98				1925.04	2053.46
				=	33%	67%					
Country 1	non-life	non cat unc.	200								12.18
		non cat vol	20								0.42
		cat. risk	250								69.62
		Interest	300								257.33
		Total	770		287.92	482.08				421.26	339.55
				=	37%	63%					
		Total group				3852.32		486.03	3366.29	3366.29	3366.29
							=	13%	87%		

Table 5a shows the result of proportional allocation: each BU gets 13% reduction. And in the last column the diversification is allocated back taking into account the contribution to the total diversification effect. As you can see the insurance risk (life and non-life) are reduced dramatically. By itself not strange: it is the task of an insurance company to diversify risks. The highest risk, interest rate risk, gets relatively less. A very high reduction (88%) can also be seen at trend uncertainty. This is because the two BU's have opposite risks (annuity and term insurance).

In table 5b the calculation is done as if the two life companies have the same type of risk:

Table 5b

Example diversification insurance group (other trend correlation between BU's, same sign of the risk)

			Capital		Diversification within BU	Stand Alone BU		Diversification group	Total Group capital	Capital after alloc. prop	Capital after alloc. marg.
Country 1	life 1	trend +	400								107.93
		level	300								29.54
		volatility	150								10.94
		calamity	100								26.27
		Interest	1000								842.30
		Total	1950		782.74	1167.26				1038.72	1016.98
				=	40%	60%					
Country 2	life 2	trend +	700								204.19
		level	600								105.45
		volatility	10								0.48
		calamity	0								0.00
		Interest	2000								1767.56
		Total	3310		1107.02	2202.98				1960.39	2077.69
				=	33%	67%					
Country 1	non-life	non cat unc.	200								11.96
		non cat vol	20								0.41
		cat. risk	250								68.37
		Interest	300								252.69
		Total	770		287.92	482.08				428.99	333.43
				=	37%	63%					
		Total group				3852.32		424.22	3428.10	3428.10	3428.10
							=	11%	89%		

As you can see the capital for trend uncertainty is much higher, but all the others are close to the results in table 5a. At least it proves that the allocation is in such that the diversification is going where it is created.

In 5c a combination of both methods is shown. The reward of being part of the group is the same for all BU's, but the allocation back to the risk types is done in a marginal way. The idea is that the risk manager in a Business unit can only manage his own risks, not the risks in other BU's.

Table 5c

Example diversification insurance group

			Capital		Diversification within BU	Marginal allocation by risk within BU		Diversification group	Total Group capital	Capital after alloc. prop group div.
Country 1	life 1	trend +	400			137.07				
		level	300			86.74				
		volatility	150			32.13				
		calamity	100			33.20				
		Interest	<u>1000</u>			<u>878.12</u>				
		Total	1950		782.74	1167.26				1019.99
				=	40%	60%				
Country 2	life 2	trend -	700			222.43				
		level	600			164.10				
		volatility	10			0.73				
		calamity	0			0.00				
		Interest	<u>2000</u>			<u>1815.73</u>				
		Total	3310		1107.02	2202.98				1925.04
				=	33%	67%				
Country 1	non-life	non cat unc.	200			85.05				
		non cat vol	20			2.90				
		cat. risk	250			168.54				
		Interest	<u>300</u>			<u>225.59</u>				
		Total	770		287.92	482.08				421.26
				=	37%	63%				
		Total group				3852.32		486.03	3366.29	3366.29
							=	13%	87%	

d. Testing the impact of correlation assumptions

In this part the impact of the several correlation factor assumptions is tested.
First the correlation factors between risk types are analysed: with what amount will the total diversification decrease in case no diversification was allowed between the two risk types? This is done by setting the correlation factor at 1 (in table 6a the 10 correlation factors with the highest impact are given).
Second the correlation factors for each risk between the BU's are tested. This is done by setting all the factors for that risk we assumed between the BU's at 1 (table 6b shows this effect).
Third the step of 0.25 is tested. Is this step not too coarse? This impact is calculated by setting the correlation factor 0.25 lower and seeing what the impact is on the total diversification (table 6c).

Table 6a impact by setting correlation factor at 1

Risk 1	Risk 2	Impact amount	Impact % total diversification
Mort. Trend unc.	Interest rate risk	709.5	27%
Mort. Level unc.	Interest rate risk	568.1	21%
Mort. Trend unc.	Mort. Level unc.	156.8	6%
P&C current cat risk	Interest rate risk	139.2	5%
P&C cur. non cat unc	Interest rate risk	98.1	4%

Table 6b impact by setting correlation factors between BU's at 1

Risk	Impact amount	Impact % total diversification
Interest rate risk	194.0	7.3%
Mort. Trend unc.	82.2	3.1%
Mort. Level unc.	56.5	2.1%
P&C cur. Cat risk	9.3	0.3%
Mort. Volatility	3.9	0.1%

Table 6c impact by decreasing correlation factor with 0.25

Risk 1	Risk 2	Impact amount	Impact % total diversification
Mort. Trend unc.	Interest rate risk	202.1	7.6%
Mort. Level unc.	Interest rate risk	157.7	5.9%
P&C current cat risk	Interest rate risk	47.7	1.8%
Mort. Tend unc.	Mort. Level unc.	40.3	1.5%
P&C cur. non cat unc	Interest rate risk	25.0	0.9%

Because of interaction between factors these factors can not be added!

Possible conclusions looking at these figures:

Appendix B

Extra attention for the interest rate risk related factors is necessary; particularly the correlation between mortality trend and mortality level needs some extra attention. Also for interest rate risk a step smaller than 0.25 needs to be analysed.

Be aware that this is only an example. In real situations, where also other risk types are involved the result will look different.

e. **How to deal with diversification at group level.**

Suppose the first BU (country 1 life) is a stand alone company. The balance sheet will look like:

Company 1	
Assets 11,167.28	Liabilities 10,000 Capital 1,167.26

As part of the group with reduction of the capital needed by 13% (allocated back from group diversification):

Company 1 as part of group	
Assets 11,019.99	Liabilities 10,000 Capital 1,019.99

It can be defended that a company as part of a group should hold the same capital as a stand alone company. In that case the group can give a commitment for the allocated group diversification. The balance sheet will look like:

Company 1 as part of group	
Assets 11,019.99 Group "securitization" 147.27	Liabilities 10,000 Capital 1,167.26

Solvency assessment for an entity that is part of a financial group

A GC WORKING GROUP 5 ISSUE PAPER

BACKGROUND

As part of its support for the EU Solvency II project, Group Consultatif has set up a number of working groups. Working group 5 is to address group and cross-sectoral issues.

This issue paper discusses some of the potential issues identified by working group 5. Many of these issues are of a non-technical character, and other issues that are of a more technical nature are described in another paper.

In this paper we assume that in Solvency II valuation of capital requirements and available assets will follow economic principles.

THE PROBLEM

Complications exist when viewing the solvency of a group of companies. Whereas the individual legal entities can always be viewed on a stand alone basis, the solvency of a group of companies is not necessarily equal to the sum of the parts, and questions arise as to what adjustments should be made to the solvency assessment of individual entities to take account of their membership of a group.

There are arguments that the group should be the primary focus of supervision. However an underlying assumption in the following is that whilst the solvency of a group as a whole is of interest, the primary consideration of regulators will still be the solvency of individual entities. Hence the importance of considering suitable adjustments to an individual entity's solvency assessment.

For the purposes of this paper the definition of a "Financial group" is a group of (regulated) entities whose primary business is financial. This would include insurance, banking and securities business. Note that in the Financial Conglomerate Directive reference is made to such groups, although here the definition demands that the group is active in more than one sector (e.g. insurance and banking). At this stage we do not comment as to what level can be considered as a controlling interest.

Of particular significance in assessing the solvency of a group, is to look at the influence of the various possible corporate structures, as well as the effect of different intra-group transactions. Two important areas of risk that need to be considered are:

- Diversification: the fact that the group is engaged in a variety of business areas in a variety of markets can mean that the overall level of risk is reduced at the group level
- Contagion: the fact that an entity is a member of a group can mean that it is exposed to secondary effects from problems in other parts of the group. An example of this being if one entity in a group involved in banking has financial problems, customers in other banks in the same group might call into question the financial stability of those banks and the result can be a "run on the bank".

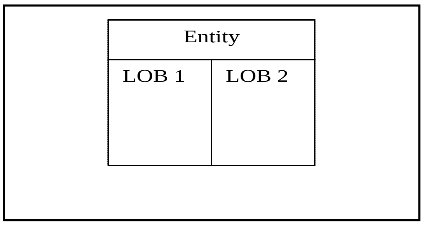
If one can ignore the various frictional effects of running a business in a number of legal entities (these effects are discussed separately), it is reasonable to expect that the assessment of solvency of a group is neutral to corporate structure of the group. However at an individual entity level this may not be true.

The assessment of solvency is further complicated when the entities within the group are not in the same sector and are thus ruled by different definitions of required solvency. This can mean that there is a lack of consistency in measuring solvency across the group.

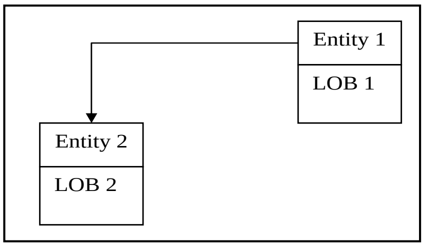
POSSIBLE CORPORATE STRUCTURES CONSIDERED

Whilst there are any number of combinations and variations, a distinction is made between four different forms of corporate structures for providing financial services in a group. The structures are illustrated by considering a group that has two lines of business (LOB)

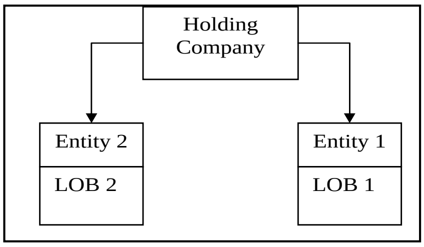
1. Integrated model: here financial services are offered within the same legal entity. The solvency of the business would then be considered as a whole, and any diversification effects between the lines of business would naturally be taken into account when assessing the whole entity.



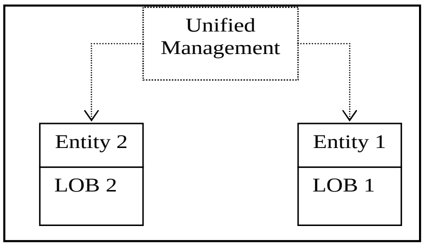
2. Parent-subsidiary model: here one regulated operating entity owns (or part owns) another legal entity with operational activities. The relationship between the entities includes a direct capital relationship. In this case the lines of business are conducted in different entities and initially the solvency will be assessed on an individual entity level, and so diversification effects between the lines of business will not be normally be included. Note that here we can also include cross shareholdings.



3. Holding company model: here a holding company without its own operational activities owns a number of subsidiaries. There is no direct capital relationship between the operating entities, but there is an indirect capital relationship. Again initially the solvency of the entities would be considered on an individual company level, and so no diversification effects between lines of business would normally be included.



4. Horizontal group model: here there is no direct or indirect capital interest but the entities have other links (such as common management). Again with the individual entities being evaluated on a stand



alone basis, any diversification effects are not captured.

5. In addition to subsidiaries branches can also be used. We observe that in pure economic terms the capital required for a parent company with a subsidiary is the same as for the parent company and a branch as the risk exposure of the combined undertakings is the same. However a key difference from a regulatory perspective is that the policyholders of a branch have automatic recourse to the asset base of the mother company whereas in the subsidiary they have no automatic rights to support from the parent. This relationship is recognised in the existing directives which place supervision of a branch of an EU regulated entity with the home supervisor of the parent of the branch. In particular a branch structure would allow full recognition of diversification effects in capital assessment. By comparison some restriction may be justified in the parent subsidiary structure following assessment of any limitations on access to capital support.

Naturally in practice a large group would have a complex combination of the above.

CONTAGION RISK

Under contagion risk we refer to the risk that an entity is impacted by events in a fellow subsidiary or the parent. As such contagion is not a risk type but a consequence of a risk event.

The classic example is the ‘run on the bank’ where events in one branch or entity lead to a loss of depositor confidence in other parts resulting in severe liquidity issues.

We believe that contagion in insurance companies is likely to be different to the banking environment. In Non-Life insurance and mortality business an insured event has to occur before a claim for payment can be made. In life assurance where policyholders may have a surrender option a ‘run’ would be possible but where surrender values are adjusted according to market values or where tax charges for the policyholder are triggered there can be mitigation effects. Where surrender values are guaranteed at high levels the ‘run’ has more potential to cause significant damage. We also note that group contracts may have delayed settlement provisions mitigating liquidity issues.

One useful survey on contagion is the Freshfields paper prepared for the Dutch regulator. We feel that contagion is not generally susceptible to additional capital requirements but should be addressed through risk management processes which are reviewed under Pillar II.

Limited liability is one tool for the management of some aspects of contagion risk. This can be used to limit the impact on group members from the events in a particular entity. This will be a comfort to regulators but where the group intends to strongly limit its support that will need to be reflected in the capital requirement of the particular entity.

ASSESSMENT OF SOLVENCY ON A STAND ALONE AND GROUP BASIS

The objective is to ascertain a target level of solvency (solvency capital requirement or SCR) and compare available risk capital (ARC) to that level. This can be done on a stand alone basis for each entity in a group, and also on an overall group level basis.

Initially each legal entity can be considered on a totally stand alone basis, i.e. as if it was not part of a group. The question is then what adjustments should be made as a result of membership of a financial group.

A standard adjustment that is made under current rules is the elimination of interest in other (financial) entities. This is to avoid "double gearing" of solvency capital. Whilst it is logical not to double count equity, a weakness of the approach is that a company may not get any credit for its ownership of "excess capital" in a subsidiary. Today other adjustments are not usually made.

The group solvency can be assessed in a number of ways. The most appealing approach is to use the consolidated accounts where all intra group transactions and balances are eliminated. Another approach is to add capital (both SCR and ARC) from the relevant entities. The danger here being that certain "consolidation effects" are missed. Another consideration is the elimination of intra group credit risks. Note that in assessing the amount of SCR at the group level, it is possible to take into account group effects such as diversification, contagion effect etc.

The same arguments can be applied to MCR (the minimum capital required). However at a group level it is uncertain if MCR plays a role. If the available capital is under MCR for a group as a whole, then it is almost certain that a number of entities within the group have had capital under MCR for some time, and this means that action will have already been taken (which may mean effective removal from the group).

When assessing group solvency it is also necessary to consider the actual ability of the group to transfer capital between entities and account should also be taken of the fact that risk capital is not necessarily easy to transfer within the group, for example some non EU entities may not be able to release capital due to local regulatory constraints.

Given that the regulators primary concern is the solvency of individual entities, it is of interest to consider how the solvency assessment of an individual entity can be related to the solvency of the group as a whole.

ADJUSTMENTS TO STAND ALONE ASSESSMENTS IF MEMBER OF A GROUP.

Whilst a regulator may start from the position of considering a legal entity on a stand alone basis, there are a number of reasons for adjusting the assessment when an entity is part of a group. These reasons include:

- Effect of diversification within the group
- Contagion issues within the group
- Access to extra capital within the group
- Commitment to supply capital to other entities within the group
- Other effects of group activities

There are a number of adjustments to the assessment of an entities solvency that could be considered:

- Elimination of investments in group entities (i.e. entities with operating activities): as previously stated this is a reduction in available capital prescribed by current rules. This could be taken as an adjustment to the ARC

  The degree of such an adjustment is a topic that would merit further consideration.

- On a pure economic approach to the balance sheet we would not expect any goodwill element. If one does arise existing rules would require its elimination. We note there is an argument that goodwill represents a value that can for example be realised on sale of that investment

- Adjustments to allow for access to capital from other entities within the group. This could also be an adjustment to ARC.

  The necessary conditions for such a credit would need to be examined.

- Adjustments to allow for explicit or implicit commitments to supply capital to other entities in the group: this would naturally be an adjustment to ARC

- Inclusion contagion risk: a natural way to make an adjustment for this would be through an adjustment to SCR. However contagion risk can be difficult to quantify and might be better considered under “pillar II”.

- Whilst in considering a group's solvency it is natural for diversification to be seen as reducing required capital when considering the position in an individual entity within a group it is more natural to consider diversification at group level as creating a form of available capital (ARC).

- Whilst the same adjustments could be made to MCR, in practice regulators might feel reticent to do this as the action takes on a more legal status/ as the MCR needs to be a straight forward formulation that can be demonstrated in courts at time of distress. If the total of MCR and technical provisions are not too high it would not be necessary to consider diversification effects outside the legal entity.

A pragmatic approach to adjusting ARC:

One pragmatic approach to adjusting the amount of available risk capital, which takes into account many of the above effects, could be to use the following formula:

$$ARC'_S = c * ARC_S + (1-c) * SCR_S * (ARC_G + SCR_T – SCR_G) / SCR_T$$

Where

ARC'_S = Adjusted available risk capital

ARC_S = Available risk capital prior to adjustment

ARC_G = Available Risk capital on a group basis

SCR_S = Solvency capital required on a stand alone basis

SCR_G = Solvency capital required on a group basis

SCR_T = Total of SCRs for individual entities, i.e. $\sum$ SCRs

c = a constant that depends on the relationship of the entity in question to the group

The extent to which group capital is taken into account is thus driven by the constant “c”. If “c” is 1 then no account is taken of group capital, if “c” is 0 then full account is taken of group capital levels. Note that the adjustment can be both positive and negative. That is to say an entity with relatively high capitalisation would be adjusted downwards to take account of an implicit commitment to supply the rest of the group with capital.

This formula would need to be adjusted when the ownership is not 100%

Note that since we would recognise group diversification effects to decrease SCR_G, it is not necessarily true that $SCR_G = \sum SCRs = SCR_T$.

OTHER GROUP ACTIVITIES THAT CAN HAVE AN EFFECT ON THE SOLVENCY OF ITS MEMBERS

There are a number of benefits of being a member of a financial group that should be taken into account when assessing solvency. Equally there are also areas of potential additional risk to consider. These effects are usually of a more "soft" character and it can be difficult to put an objective measure on them

Beneficial effects to consider

1. A financial group often has its own "oversight" function(s), which means that the quality in areas such as audit, risk management etc has benefited, both as a result of having had a "second pair of eyes" and also experience from other markets/ sectors

2. Often there is coordination of liquidity requirements within a group (for example through the use of common cash accounts etc), and through this, access to liquidity is improved for the individual entity

3. A coordination of capital acquisition activities can provide benefits in terms of speed, availability and cost of new capital. Also capital released within the group is more readily available to meet the needs of other parts of the group.

   In a well managed group this can also reduce pressure on entity management to find a use for idle capital.

4. As the groups reputation is important an individual entity can find that it has a great deal of support from the group in avoiding and solving reputational problems

Areas of potential increased risk

1. There can be pressure to accept terms and conditions to support wider group interests

2. There can be pressure to maximise exposure for a given level of capital in order to support a group capacity and limit costs associated with capital movements or risk transfer

3. There can be exposure to fellow subsidiaries and the entity management may not have the access to the same readily available credit ratings as it would when dealing with external parties. Effective group supervision would mitigate this.

4. In a wide spread group, management might have limited relevant insurance experience, but at the same time exert strong influence on the entity level management.

5. The existence of a group with deep resources may encourage local management to be more risky.

We might expect these and other issues to be dealt with by qualitative requirements on management in general and risk control in particular.

INFLUENCE OF INTRA GROUP TRANSACTIONS

Group structure will impose relationships depending on company law and possibly other local legal requirements. In addition there are a number of intra group transactions that create capital obligations

There are a number of forms of intra group transaction to consider. Many of these transactions can and are used to transfer the “group effects” described above around a group.

1. Capital: pure capital can be paid from one entity to another. In this case the relationship is quite straight forward, and little differs from a stand alone entity

2. Forms of capital guarantee: here a company guarantees a supply of capital. Under current rules for insurance companies, to a limited extent one is allowed to use “subscribed but not paid” capital as part of ARC and this is one form of guaranteed capital. There are a number of forms in which capital can be guaranteed, and a decision needs to be made as to how water tight the guarantee needs to be in order for it to be included in the ARC

3. Internal reinsurance: internal insurance is one way of explicitly taking advantage of diversification effects, at least as far as underwriting risk is concerned. By swapping underwriting risk between entities, both entities can benefit from a diversification risk effect. A problem with internal reinsurance, is that if an entity is considered on a stand alone basis the credit rating of the reassurer comes into play thus introducing extra risk that does not exist on a group basis (where the transaction is eliminated on consolidation)

4. Contingent loans (subordinated debt etc): this another form of capitalisation, which can under certain circumstances be considered as ARC

5. There are other forms of capital transfer: in some jurisdictions, for example in Sweden there is “concern contribution” – essentially a tax free transfer of capital.

6. Internal cross charging: although there are often rules covering transfer pricing, one way of “transferring” capital is to adjust the amount being charged for goods and services between entities in a group.

When considering intra group transactions, care needs to be taken to ensure that there is not any “artificial capital creation”. This is particularly important where there are non-regulated entities and/ or entities outside the EU. Examples being: (a) a holding company that borrows heavily to capitalise its subsidiaries; (b) an offshore reinsurance entity with no risk capital.

It should be noted that there is possible over and under capitalisation in various entities within a group.

Whilst it is logical to eliminate multiple gearing of capital there is a case for making an allowance for part of any excess capital (ARC-SCR) in a subsidiary.

All of the above problems can at least be partly addressed using the adjustment formula proposed above

As mentioned above, consideration needs to be made regarding how one takes into account the credit risk between group companies

Non-EU holdings can cause problems, for example some jurisdictions might require higher levels of solvency capital, which means that group solvency capital is more difficult to access.

INFLUENCE OF CORPORATE STRUCTURES

Special consideration needs to be taken depending on the corporate structure

1. parent-subsidiary (or intra group cross shareholdings)

    a. In this case the ownership of capital needs to be considered carefully. Whilst there is a logic that the investment in risk capital in a subsidiary should be excluded from ARC in the parent, it can be argued that the investment does have a economic value as evidenced if the subsidiary is sold to a third party

    b. The position is even more difficult when a cross shareholding is involved. Here is easy to see that an amount of illusionary capital is generated. For example consider two companies with ACR of 100, and each owning 50% of the other. In this case if the companies were merged the total capital available would be only 100 and not 200.

    c. In a true parent subsidiary relationship one also needs to consider the fact that moving capital “upstream” is often more difficult than moving it downstream. For example putting more capital into a subsidiary is straight forward, but paying capital to a parent can be difficult given restrictions on dividend payments etc

    d. Cross shareholdings with non EU entities can be particularly difficult to monitor and analyse

2. Holding company

    a. A holding company is often not regulated and it is possible to under capitalise the holding company. This can for example be done by heavy borrowing at the holding company level

    b. Transfer of capital within the group can be difficult. Examples of causes; tax, regulatory constraints. Winding up rules giving protection to certain classes of creditor and this may vary by country

    c. The “distance” (i.e. length of ownership chain) can influence the extent to which one can expect capital support.

d. There can be a difference for an individual entity between its relationship to another entity where there is a direct "parental" relationship as opposed to entities with the same parent.

3. Horizontal group

a. There is no direct capital relationship and so the extent of the relationship is not clear. In particular, with no explicit capital relationship, the ability to access group capital will be weak. And the contagion risk may be less due to the separation although common control speaks against this.

b. If a group starts to experience financial difficulties in part of its operations, it is often quite easy for entities that are healthy to leave the group without any penalties, which can make matters worse for the entities left (both in terms of access to capital and loss of reputation) This can also arise in other group structures where the stronger components of a group can be sold to distance them from the weakened elements.

FRICTIONAL COSTS

There are a number of frictional costs

1. Tax: transfer of capital is not always possible without incurring a tax penalty

2. Dividend restrictions: transfer through dividends can be restricted for example by the regulators

3. Currency losses: there are costs involved in currency exchange and indeed there can be restrictions of currency exchange that need to be considered

AGGREGATION INSURANCE, BANKING AND INVESTMENT OPERATIONS

There are a number of difficulties caused by aggregating various financial services businesses, in particular:

1. The basic approach in Basel II is top down, rather than the bottom up approach envisaged for insurance.

2. The time horizons considered for banking are short in comparison with insurance. Whilst there are good reasons for this, it makes comparisons of solvency positions difficult

REGULATORY CONSIDERATIONS

Since the supervision of a group of companies will differ, some attention needs to be paid to the effect of the supervision process on a financial group, particularly if it is operating in a number of jurisdictions.

1. There can be different legal and capital structures used in various markets which can make comparisons difficult

2. Despite the efforts of EU legislators, there can be a lack of consistency in how capital is defined

3. The extent of information required by the regulators can vary.

4. It is unclear how the cooperation between regulators can impinge on the solvency assessment

PRINCIPLE OF GROUP SUPERVISION

Supervision in the EU started with the solo entity and only more recently were measures on group aspects added. Indeed the procedures are still being tuned.

The blue print for Solvency II adopts the existing structures. However we feel that consideration of how the system can recognise diversification effects at group level and group level models will lead to a need to reconsider the roles of group lead supervisor and local supervisor.

A simple view would be that if a group has adequate capital in total and appropriate risk management procedures the regulation of each entity can be reduced. However we recognise that many regulators and their Finance Ministries will be uncomfortable with the reduction in local security this implies. It is however a logical conclusion of the single market concept.

We note that the Capital Requirements Directive (Article 69) allows Member States to regulate capital at the parent level where the group is within a single state and meets certain conditions.

CONCLUSIONS

The following issues need to be considered:

1. There is potential for diversification effects whenever blocks of business are aggregated. In particular the economic capital for an entire group can have material diversification effects compared to the sum of the constituent entities.
   Solvency II should consider to what extent this economic impact can be reflected in capital requirements...

2. Groups present additional risk compared to solo entities but can also bring beneficial effects. A balanced view is needed.

3. A system giving credit for group level impacts will need to establish principles to guide the assessment of intra group risk transfer and intra

group capital commitments. Principles are required rather than rules as an exhaustive list of such instruments is likely to be very complex and become outdated.

4. Contagion is a valid concern but not necessarily the same as for banking. Many aspects are best dealt with through effective risk management procedures and assurance rather than arbitrary capital requirements. Some such as damaged reputation leading to surrenders needs careful assessment of the underlying technical position such as guarantees, the mitigation measures available to the entity (e.g. deferment or market value adjustments) and what is already captured in any capital requirements driven by discontinuance shocks.

5. The arrangements that apply for wholly with in the EU groups need evaluation for the consequences of non EU entities as group members or parent.

6. Conglomerates open the issue of aggregation of insurance, banking and investment operations, and non regulated entities. Particular problems include:

    a. Banking tends to follow a top down approach whereas Insurance tends to view solvency bottom up

    b. Basel II measures the time horizon for solvency purposes in terms of days, whereas insurance has a time horizon of years (or at least one year)