Charity over Ultimate Forward Rate

The Ultimate Forward Rate is an important concept in Solvency II and in other solvency regulations. Especially for contracts with a longer maturity, as long as market consistent valuation is required, it is essential to know how to discount cash flows. On one hand there are no market interest rates for very long durations. On the other hand many contracts are long-dated and their valuation is very dependent on discount rates. For undertakings an essential problem is how to hedge interest rate risk and the choice of this rate, not replicated from actual market rates, is extremely important. Overall it will of course be important to choose a value and methodology that does not put policyholders in jeopardy with insurers covering their obligations recklessly.

This text does not attempt to give the reader an exact answer on how to set the Ultimate Forward Rate. Instead, it attempts to clarify the concept and additionally give some general guidelines on how to evaluate different proposals for the methodology to fix the Ultimate Forward Rate (UFR) to one or the other certain value. As we are talking of a value a long time in the future where no reference to actual markets can be made we have to accept the fact that the value is to some degree arbitrary.

In the context of the UFR there are at least four essential problems:

- what is the level of the UFR,
- at what time point do we expect the UFR to be reached,
- until what time point do we expect markets to give us reliable discount rates/risk free rates, i.e. what is the last liquid point (LLP), and
- how does the interest term structure develop from the LLP until UFR is reached – i.e. what is the interpolating method used in between?

In Solvency II some answers to the questions above are fixed fairly strongly in the framework directive or the delegated acts, whereas other concepts are more easily changed with a mandate given to EIOPA. In the following all concepts and their interconnectedness are discussed as we feel it essential to look at the overall setting. Where some concepts might be easier to change we must however evaluate also whether they are consistent with assumptions taken in other areas, irrespective of whether it will be possible or not to change the other values.

1. The concept of UFR as a forward rate
   The Ultimate Forward Rate is defined as a one year risk free rate for an investment at some future time point. For shorter time periods the implied forward rate can be calculated when we have information on two consecutive interest rates. The forward rate is then one year interest rate that can be replicated with no arbitrage with these rates, i.e. an investment of n+1 years must equal the investment of n years continued by a one year investments with the yield of the forward rate. Depending on the way interest is credited there are slightly different ways of calculating the value of the forward rate resulting into slightly different answers but these different methodologies are not important as regards the problem we are now talking about. Now it is only essential to know that as long as there are deep and liquid markets there are methods of arriving at a well-defined value of the forward rate.
   Here we start with a continuous (but deterministic) model. If we think of a bond with nominal value of 1 and we denote by P(t,T) the price at time t of a zero-coupon bond that matures at time T, then the spot rate R(t,T) at time t for maturity at time T is defined as the yield to maturity of this bond:
   $$\mathrm{R(t,T)} = \frac{\log P(t,T)}{T-t}.$$

Forward rate F(t, T, S) at time t, between T and S (t ≤ T < S), using this notation, is:

$$\mathrm{F}(t,T,S) = \frac{1}{S-T} * \log\frac{P(t,T)}{P(t,S)}$$

Again with this notation the ultimate forward rate UFR(t) at time t (as S = T+1) is given by

$$UFR(t) = \lim_{T\to\infty} \log\frac{P(t,T)}{P(t,T+1)}$$

Two important things to note are:

- there is no single way of deriving the UFR from markets as the UFR as such cannot be an exact market consistent value because there are no markets for extremely long maturities, and
- the UFR is a forward rate that is not easily tied to market rates or forward rates implied by market rates giving values only for maturities extending at most to about twenty years.

As there is no exact way of deriving an UFR we need to develop a methodology to set the rate. All methodologies are to some extent arbitrary and they all have their shortcomings. There are at least three ways of deriving forward rates:

- taking observed rates of longest maturities and calculating longest observable forward rates from them and then relying on a chosen convergence to fix the UFR,
- starting from macroeconomic assumptions connected to inflation and real growth and deriving the UFR that is consistent with these, or
- using a stationary distribution based on, e.g. Cox-Ingersoll-Ross, estimated with ML or QML, calculating its moments and deriving an UFR based on them (METHOD PROPOSED BY LUIS ALVAREZ, MUST AT LEAST BE MADE MORE EXACT, IF INCLUDED).
    a. Forward rates from observed longest maturities
    This could be formulated in different ways but essentially the problem can be defined as:

    $$\mathrm{e}^{\mathrm{F}(0,\mathrm{S},\mathrm{S}+1)} = \mathrm{E}\left[\mathrm{e}^{\mathrm{R}(\mathrm{S},\mathrm{S}+1)} \middle| \widetilde{F_0}\right]$$

    where $\widetilde{F}_t$ represents the information available at time t. It is easy to verify that $e^{R(S,S+1)}$ and $e^{R(S+1,S+2)}$ must now be uncorrelated, which probably is not intuitively true in reality. But in any case, this formulation can give some guidance on setting forward interest rates.
    b. Using macroeconomic assumptions
    Long-term expectations of interest rates can be decomposed into two components. The first of these is based on expectations of future short-term interest rates, at least on average level. In addition to this investors require a premium for the risk of a long-term investment which forms the second component.
    Expectations of interest rates are driven by inflation expectations and expectations of future economic growth. Inflation expectations depend to a large extent on the policy of the central bank. Future economic growth on its part is basically tied to the increase in work force and productivity gains. Very exact and reliable estimates of future interest rates are of course impossible to come by based on these factors, especially when we talk of really long time scales.
    The second component is the risk premium required by investors. It seems natural to think that this risk premium increases with the time horizon getting longer. This reflects the fact that there is more uncertainty both with respect to future inflation and future economic growth rate.
    There has been discussion on what are the main factors in the decline of interest rates during last years. It seems that the so-called standard models have suggested

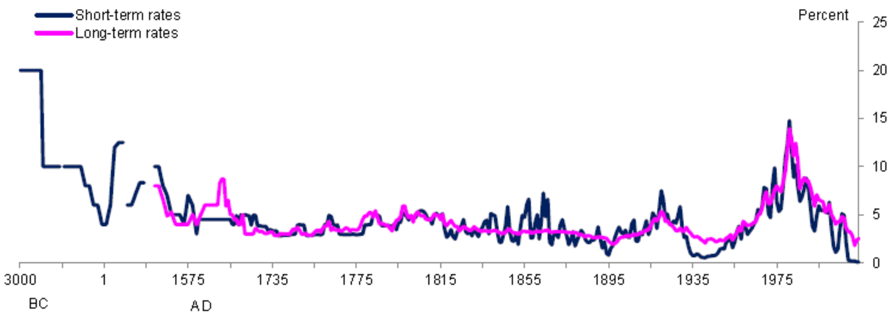
that rates have gone down mainly because the risk part has gone down. However, some recent studies [see Bauer et al.] indicate that:

- the decrease is mainly due to lower estimates on the level of inflation and future economic growth, and
- actually the premium for risk has even increased – and also the risk premium has been countercyclical, i.e. rising in recessions and falling in expansions.

With all these methods there will be a value for the UFR. This can still be evaluated by deriving macroeconomic variables that are consistent with the value of the UFR. With the second model above the values of these macroeconomic factors are easy to set. With the first and third model one essentially needs to see what would be the values in the second model leading to these values.

Another reason for caution with the first approach is whether market rates generally forecast market consistent forward rates far in the future. This might be especially true in current economic environment where one could say that interest rates are exceptionally low. For example, it has been said that interest rates are now lower than what they have ever been during the last 5000 years.

Chart 5: Short and long-term interest rates



Sources: Homer and Sylla (1991); Heim and Mirowski (1987); Weiller and Mirowski (1990); Hills, Thomas and Dimsdale (2015); Bank of England; Historical Statistics of the United States Millenial Edition, Volume 3; Federal Reserve Economic Database. Notes: the intervals on the x-axis change through time up to 1715. From 1715 onwards the intervals are every twenty years. Prior to the C18th the rates reflect the country with the lowest rate reported for each type of credit: 3000BC to 6th century BC - Babylonian empire; 6th century BC to 2nd century BC - Greece; 2nd century BC to 5th century AD - Roman Empire; 6th century BC to 10th century AD - Byzantium (legal limit); 12th century AD to 13th century AD - Netherlands ;13th century AD to 16th century AD - Italian states. From the C18th the interest rates are of an annual frequency and reflect those of the most dominant money market: 1694 to 1918 this is assumed to be the UK; from 1919-2015 this is assumed to be the US. Rates used are as follows: Short rates: 1694-1717- Bank of England Discount rate;1717-1823 rate on 6 month East India bonds; 1824-1919 rate on 3 month prime or first class bills; 1919-1996 rate on 4-6 month prime US commercial paper ; 1997-2014 rate on 3month AA US commercial paper to non-financials. Long rates: 1702-1919 - rate on long-term government UK annuities and consols; 1919-1953, yield on long-term US government bond yields; 1954-2014 yield on 10 year US treasuries.

(Speech of Andrew G Haldane, 30 June 2015)

2. The assumptions behind the UFR

There have been different approaches on the setting of the UFR. Solvency II approach bases itself mainly on the second approach above, i.e. on macroeconomic factors. Another approach is taken at least in the Netherlands for pension obligations, i.e. the starting point there is to look at long-dated bonds (and the level of the UFR is then 3,9 % - see Langejan et al).

Solvency II UFR starts from ECB inflation target which is 2 %. To this it adds a real growth rate of 2,2 %. This methodology, at least not explicitly, does not talk of any risk premium added to the sum of these.

There has been criticism on the level of 4,2 % as the UFR being too high. The main motivation for this criticism has been the fact that interest rates have fallen to quite a lot lower levels and insurers do not easily get returns reaching the level of the current UFR. Also, in some contexts, like the Dutch UFR mentioned above and also the values used by the IAIS, there have been lower UFR's in use. Also ESRB (Vulnerabilities issue note of 8 June 2015) has stated that the real interest rate of 2,2% is likely too optimistic by 0,5-1,0 percentage points.

On the other hand it has been said that the UFR should not be too sensitive to changes in the market. Another issue to be checked is what the risk premium mentioned above should be like. Additionally and in all approaches one should also look at macroeconomic assumptions made elsewhere. It would probably not make much sense to compel insurers into using lower assumptions on growth than what is used elsewhere in the society – would it make much sense to require insurers to stay solvent while everything around collapses?

3. Last liquid point – do we have liquid markets until LLP?
   Siegbert – could you write here what is cast in stone with LLP?

   Directive 2009/138/EC Article 77
   (Extrapolation of the relevant risk-free interest rate term structure):

   > *The determination of the relevant risk-free interest rate term structure referred to in Article 77(2) shall make use of, and be consistent with, information derived from relevant financial instruments. That determination shall take into account relevant financial instruments of those maturities where the markets for those financial instruments as well as for bonds are deep, liquid and transparent. For maturities where the markets for the relevant financial instruments or for bonds are no longer deep, liquid and transparent, the relevant risk-free interest rate term structure shall be extrapolated.*
   >
   > *The extrapolated part of the relevant risk-free interest rate term structure shall be based on forward rates converging smoothly from one or a set of forward rates in relation to the longest maturities for which the relevant financial instrument and the bonds can be observed in a deep, liquid and transparent market to an ultimate forward rate.*

   Directive 2009/138/EC and the Delegated Regulations set out for the determination of the LLP, the application of DLT requirements and for the Euro a specific recital regarding the residual volume of bonds meeting DLT requirements (aka the '*residual volume criteria*'). For the Euro, the method is precise except for the very specific market data to be used as input.

   Recital 21 of the Delegated Regulation defines the ***residual volume criterion*** to calculate the LLP. The residual volume criterion is used to derive the LLP for the euro only. For that currency, it gives an LLP of 20 years. The 'residual volume criterion' considers all bonds in the market, including corporate bonds. Having computed the outstanding bond volume for each maturity, the sum of the outstanding bond volumes for all maturities ≥M is computed. The smallest maturity M for which that sum drops below 6% is considered to no longer meet the DLT criteria. For the Euro, this gives an LLP of 20 years.

For all other currencies, the LLP has been chosen according to the results of the DLT assessment. It is the longest maturity for which risk-free interest rates can be derived from DLT markets.[1]

The LLP is considered to be generally stable over time, which means that once the first full DLT assessment is done, it is not necessary to pursue a continuous repetition of the full assessment, but, rather an appropriate monitoring of financial markets. It is thus expected that the update of the LLP will be carried out at a lower frequency than the publication of RFR curves (e.g. yearly basis).

4. How does the convergence from LLP to UFR work?
Siegbert – same thing here?
COMMISSION DELEGATED REGULATION (EU) 2015/35 of 10 October 2014
Article 46 **Extrapolation**
1.The principles applied when extrapolating the relevant risk free interest rate term structure shall be the same for all currencies. This shall also apply as regards the determination of the longest maturities for which interest rates can be observed in a deep, liquid and transparent market and the mechanism to ensure a smooth convergence to the ultimate forward rate. ....

27. The Omnibus II Directive explicitly reflects for the euro a convergence period of 40 years and a LLP of 20 years, which is equivalent to assuming that the forward rate will be close to its ultimate level from 20+40=60 years maturity onwards.
28. For currencies other than the euro, the convergence point is the maximum of (LLP+40 years) and 60 years. This method is considered as the most stable, least influenced by expert judgement and also the one with lowest impact on the level playing field between market participants[2].

6. Extrapolation and interpolation
128. The parameters used for the extrapolation, except for the financial market data itself, will be assumed to be stable. This includes

- the ultimate forward rate (UFR),
- the convergence tolerance, the convergence period,
- the last liquid point (LLP), and
- the support of the selected extrapolation method, i.e. the maturities considered to meet the DLT requirements.

Changes in the DLT assessment measures will not necessarily translate into changes in the parameters used for extrapolation, the LLP or the support of the selected extrapolation method. Therefore, the DLT assessment will be less frequent than the publication of the risk free interest rate term structures.

[1] EIOPA-BoS-15/035; 7 December 2015
Technical documentation of the methodology to derive EIOPA's risk-free interest rate term structures

[2] EIOPA-BoS-15/035; 7 December 2015
Technical documentation of the methodology to derive EIOPA's risk-free interest rate term structures

129. EIOPA will publish the interest rates for integer maturities from one year maturity onwards.

Extrapolation method

24. The interpolation, where necessary, and extrapolation of interest rates has been developed applying the Smith Wilson method.
25. This method is of course not the only one for the extrapolation of the interest rates. In the same manner other methods have their pros and cons, the Smith Wilson method also has its own features.
26. The Smith Wilson method has been applied during the last years of the development of the Solvency II framework, and in particular in the Fifth Quantitative Impact Study (QIS5) and in the Long Term Guarantees Assessment (LTGA) that has underpinned the political agreement of the Omnibus II Directive.

6.E. Convergence point

140. The convergence point is the maximum of (LLP+40) and 60 years.
141. The parameter alpha that controls the convergence speed, is set as the lowest value that produces a curve reaching the convergence tolerance of the UFR by the convergence point. The convergence tolerance is set as 1 bp.

5. Should a stress be applied to the UFR?
   This will be fairly qualitative discussion. Although, as stated above, the UFR should not be too sensitive to market fluctuations. Why should one then stress the UFR?
6. How is the UFR connected to other elements of S II, for example to the cost of capital?
   Siegbert, do you have a clear view on this?

COMMISSION DELEGATED REGULATION (EU) 2015/35 of 10 October 2014

Article 47 **Ultimate forward rate**

1.For each currency, the ultimate forward rate referred to in paragraph 1 of Article 46 shall be stable over time and shall only change as a result of changes in long-term expectations. The methodology to derive the ultimate forward rate shall be clearly specified in order to ensure the performance of scenario calculations by insurance and reinsurance undertakings. It shall be determined in a transparent, prudent, reliable and objective manner that is consistent over time.
2.For each currency the ultimate forward rate shall take account of expectations of the long-term real interest rate and of expected inflation, provided those expectations can be determined for that currency in a reliable manner. The ultimate forward rate shall not include a term premium to reflect the additional risk of holding long-term investments.

Article 77 (5) of the Directive has defined the Cost-of-Capital rate as the rate used in the determination of the cost of providing the amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof (the risk margin).

The Cost-of-Capital rate used shall be equal to the **additional rate, above the relevant risk-free interest rate,** that an insurance or reinsurance undertaking would incur holding an amount of eligible own funds, equal to the Solvency Capital Requirement necessary to support insurance and reinsurance obligations over the lifetime of those obligations.

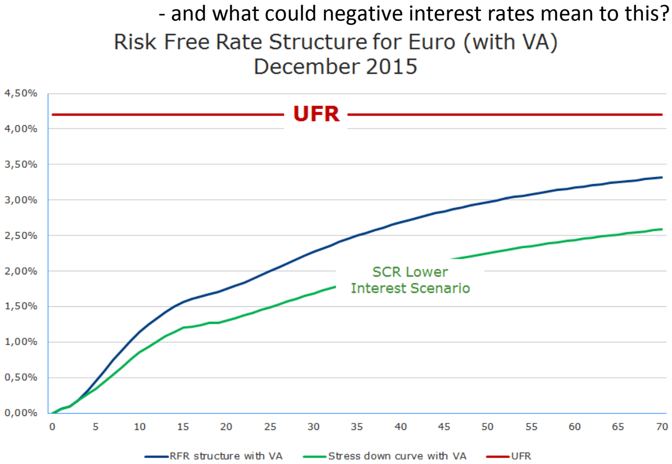
According to Article 39 of the Delegated Regulation, the Cost-of-Capital shall be assumed to be equal to 6 %.

How was the cost of capital of 6% determined?[3]

The risk measure of the SST (Swiss Solvency Test) is expected shortfall on a 99% level of confidence. This corresponds approximately to a 99.6% to 99.8% Value at Risk which implies a strong BBB- rating. For A or AA rated companies, cost of capital is in the range of 3% to 4.5% over risk-free.

For a BBB company, the cost is slightly higher, so that 6% over risk-free was chosen.

7. How similar is the S II UFR with the concept with the same name in the IAIS framework? Christoph, maybe you know how this works?
8. What is the current curve actually like and what would happen if UFR is changed?
   - and what could negative interest rates mean to this?

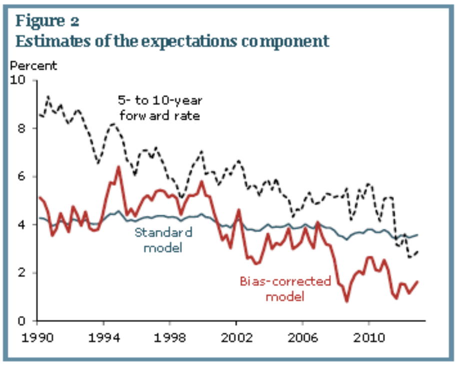


This is taken from an Insurance Europe presentation – probably needs to be replaced by and own chart.

Comment [SB1]: Similar curve is contained in the Technical Documentation EIOPA-BoS-15/035 7 December 2015 Technical documentation of the methodology to derive EIOPA's risk-free interest rate term structures p. 106, in the context of Smith Wilson method

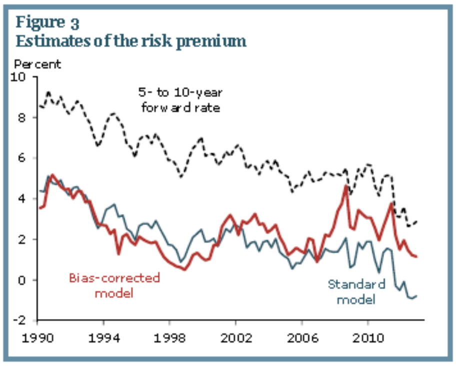
I have not been able to copy it (see last page for the result of my attempt). I will try to come to a better solution.

This one from Bauer et al could be used:



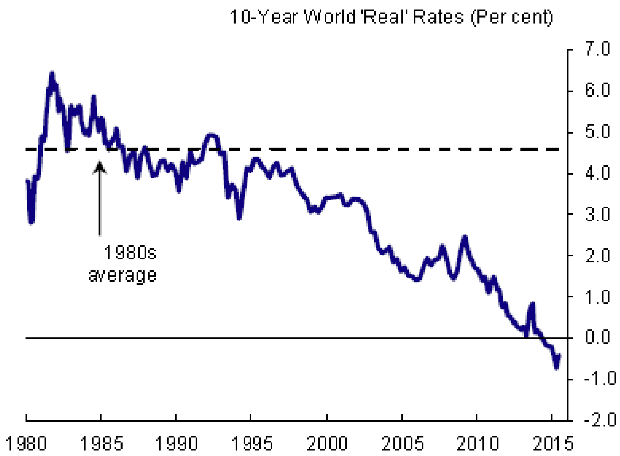
[3] The Swiss Experience with Market Consistent Technical Provisions - the Cost of Capital Approach Federal Office of Private Insurance, March 28, 2006

And also this:



Maybe also this chart from Haldane is of use?

Chart 4: Global real rates



Source: King and Low (2014); Bank Calculations. Notes: The 'World' real rate taken from King and Low (2014) and is based on the average 10-year yield of inflation-linked bonds in the G7 countries (excluding Italy). UK rates have been adjusted to take account of the RPI-CPI wedge. Data availability means that US yields are only included from 1997 so the UK provides most of the historical back-run.

(Speech of Andrew G Haldane, 30 June 2015)

References

Andrew J.G. Cairns, Interest Rate Models, An Introduction, Princeton University Press, 2004

Bauer, Michael D, Rudebusch, Glen D., What Causes the Decline in Long-term Yields? FRBSF Economic Letter, July 8, 2013

Langejan, T.W., Ewijk, C. van, Nijman, T.E., Pelsser, A.A.J., Sleijpen, O.C.H.M., Steenbeek, O.W., Advisory Report of the UFR Committee