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SPONSOR SUPPORT VALUATION AND MARKET CONSISTENCY

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Background

The recent discussions and proposals on forming holistic balance sheets for IORPs have been explicit in their desire for a 'market-consistent' valuation basis to be used. However, some of the proposed methods for valuation of sponsor support have not been aligned with the basic properties of market-consistent valuation. This short note briefly discusses the fundamental properties of a market-consistent valuation approach and discusses how sponsor support valuation methods can be aligned with this property.

The discussion is deliberately non-mathematical. Some suggested reading is provided at the end of the note that can provide a more rigorous treatment of the topic.

What do actuaries mean when they say "market-consistent"?

It is worth noting that European actuaries have been using the terminology of market-consistency extensively in actuarial practice over the last ten years or so in life insurance business. Market-consistent liability valuation is a core element of the emerging Solvency II regulation, and has been part of UK insurance regulation since 2004. It is also used in some areas of global life insurance financial reporting (for example in parts of US GAAP; MCEV).

In these contexts, market-consistent valuation almost invariably involves the following actuarial practice:

- Use a risk-neutral stochastic model to project the cashflows of the asset or liability (in insurance, this is usually liabilities from long-term savings products such as with-profits policies). Note the model may not need to be implemented using a simulation model - an analytical valuation of the instrument could be derived from the specified stochastic process. But in practice, the life insurance liabilities are sufficiently complex that an accurate analytical valuation is not easily found and so simulation models are required.

- The projected risk-neutral cashflows of the instrument are discounted using risk-free interest rate term structures. The starting interest rate term structure could be observed from government bonds, but interest rate swaps are used in some circumstances. The long-term nature of the liabilities means that the part of the yield curve that can be observed from market prices usually needs to be extrapolated to longer maturities. There may also be an adjustment to the observable risk-free term structure that is made to allow for the illiquidity of the insurance liabilities.

- The stochastic model's volatility assumptions are calibrated to market prices that are sensitive to volatility such as equity options and interest rate swaptions. Again, the available market prices may not provide a term structure of volatility assumptions that is sufficiently long-term to be directly applicable to the insurance cashflows that are being valued. So again some extrapolation method is required.

The limited availability of relevant market prices means that, in essence, the actuary is trying to build a model that is 'not market-inconsistent' instead of finding the unique 'market-consistent' model. In

order to demonstrate that the model is not market-inconsistent, a series of quantitative tests of the model output would be produced that show that the model produces valuations for elementary assets such as bonds that are consistent with the observed or assumed market prices.

Theoretical background of market-consistent valuation

For cashflows that do not vary with market risk factors such as equity risk or credit risk, market-consistent valuation simply involves calculating the expected cashflows of the instrument and discounting using a risk-free interest rate term structure that is observed in market prices of government bonds or interest rate swaps.

Market-consistent valuation gets more complex when the cashflows of the instrument are stochastic, particularly when the resultant cashflows are a function of market risk factors such as equity returns, credit spreads or changes in interest rates. In this case, the valuation of the cashflows must be risk-adjusted.

If the cashflows have a simple linear dependency on such market risk factors, then the risk adjustment is theoretically straightforward: we can calculate the (real-world) expected cashflows and discount them using a risk-adjusted discount rate that can be derived from the Capital Asset Pricing Model or one of its extensions.

In the case where the cashflows have a non-linear dependency on the market risk factors, then the valuation calculation becomes a lot harder. The non-linearity means that the 'Beta' of the instrument will not be constant through time, and it will vary continuously and stochastically as a function of how the market risk factors behave. As a result, we cannot directly apply CAPM-type logic to the market-consistent valuation.

But the work of the early 1970s of Black, Scholes and Merton pointed the way to a more general valuation method that could also be easily applied to non-linear cashflows. One of the general insights that emerged from this work was that a mathematically simpler approach to finding the market-consistent value was to risk-adjust the probability measure that was used in the valuation, instead of risk-adjusting the discount rate. Specifically, a risk-free term structure exactly like the one that is used for valuing non-risky cashflows could be used in the valuation of risky non-linear cashflows by making an appropriate change to the probability measure that the cashflows are projected in: that is, by projecting in a risk-neutral world where all risky assets have an expected return equal to the risk-free rate, the market-consistent value can be assessed as the expected value of the cashflow discounted at the risk-free rate.

This risk-neutral concept is purely a mathematical convenience. The validity of the resultant valuation does not rely on the risk-neutral condition being 'true'. It is simply more mathematically convenient than finding the market-consistent discount function to apply to non-linear cashflows. Indeed, we can turn this logic on its head and use a real-world probability measure in the valuation, and use the ratio of risk-neutral to real-world probability densities to adjust the discount function (at every part of the probability space in which the cashflows can arise). This is called the State Price Deflator approach. It will give exactly the same answer as the risk-neutral approach. Actuaries have at times been attracted to this approach as it uses real-world probabilities. But the vast majority of market-consistent valuations undertaken by actuaries around the globe today use the risk-neutral

measure. The risk-neutral approach gives the same answer as the State Price Deflator approach, is arguably easier to implement, and is usually more familiar to regulators, auditors, senior managers, etc.

Implications for valuation of sponsor support

There are two key points that can be taken from the above discussion of the technical background to market-consistent valuation: loosely speaking, any probability measure can be used in market-consistent valuation, providing a consistent discount function is used; the simplest approach is generally to use a risk-neutral probability measure, as then the consistent discount function is simply the risk-free interest rate.

In actuarial practice in insurance, all market-consistent valuations will respect the above paragraph. Some of the proposed methods for market-consistent sponsor support valuation do not. In particular, some methods propose to use real-world credit default probabilities to estimate future sponsor support cashflows, and then discount those cashflows using risk-free rates.

There are three resolutions to this issue:

1. Be explicit that the valuations are not intended to be market-consistent.
2. Use risk-neutral probabilities in simulating sponsor support cashflows.
3. Risk-adjust the discount rates that are applied in valuing the sponsor support cashflows.

The first one is self-explanatory. The second approach is theoretically straightforward: risk-neutral default rates can be derived from the corporate bond prices or credit spreads of sponsors (together with an assumption about the expected recovery rate of the bond). For sponsors where no traded debt prices are observable, some form of mapping of the sponsors' riskiness to sponsors with traded debt would be necessary.

The third approach would give the same answer as the risk-neutral method but would involve more technical complexity. For this reason I would recommend not currently pursuing this approach.

Suggested Further Reading

Financial Calculus, by Baxter & Rennie (1997). This book provides an excellent and accessible introduction to the mathematics of risk-neutral valuation.

Market-Consistency: Model Calibration in Imperfect Markets, by Malcom Kemp (2009).

Extrapolation of Market Data, by CRO Forum (2010). This paper gives a good overview of the extrapolation challenges that arise in market-consistent valuation of very long-term cashflows.

Modern Valuation Techniques, by Jarvis, Southall and Varnell (2001). This is a UK actuarial paper that discusses the State Price Deflator approach to market-consistent valuation.