Duration Matching in Insurance

Typically, an insurance contract inhibits an imbalance between cash-inflows and cash-outflows. Where usually cash-inflows occur in the beginning of the contract term, the cash-outflows are expected later in the term or even after the defined contract termination. This cash mismatch creates interest risk, when discounting is applied, as premiums have to be invested to gain interest on financial markets.

For most accounting purposes, although insurance contracts are seen as accounting unit their rights and obligations are not split in assets and liabilities, but are valued in a single insurance liability by the difference of the present value of cash-outflows (benefits and costs) and the present value of the cash-inflows (premiums). That is because inflows and outflows are seen as fully interlinked, such that there is no outflow without an inflow and vice versa. Accounting in such cases allow for netting and this results in the classical actuarial valuation of insurance contracts. It is important to say that some accounting principles as for instance those for Solvency II allow for negative liabilities if the present value of premiums exceeds the present value of benefits and costs.

In managing interest rate risk the concept of duration plays an important role. Roughly speaking, the duration describes the interest weighted life span of a series of cash-flows. The formula goes as the inner product of the discounted cash-flows with a run-time vector divided by the present value of the cash-flows. There are a lot of modifications for the duration available in textbooks as well as in business practice, but the main concept remains the same.

One of the main critical issues with the concept of duration in the context of insurance is the problem of being well-defined. As the concept of duration implies division by the present value, it must be ensured that the present value never becomes zero. To ensure this for every single point in time during the contract term it is required that all single cash-flows either are strict positive or strict negative. If the signs of the cash-flows alternate you simply can find for each discounting vector a point in time where the present value becomes to be zero and therefore duration is not defined.

In classical actuarial theory such a point in time can be found at least at the starting point of the insurance contract as premiums are calculated using the actuarial equivalence principle which requires that present value of benefit and costs equals the present value of premiums, thus the present value of the cash-flows is set to zero.

If a mathematically not defined duration would be interpreted as infinite duration the interest rate risk at that point in time also would be infinite for that contract. So, it is worthwhile thinking whether the basic netting principle in accounting of premiums and benefits is useful for risk management purposes.

At the moment duration mismatch management between insurance contracts and assets follow the accounting approach, which is

$$\text{Mismatch}_{acc} = \text{DUR}_{benefits+costs-premiums} - \text{DUR}_{assets}$$

It is noted that the above formula is not equal to

$$\text{Mismatch*}_{acc} = (\text{DUR}_{benefits+costs} - \text{DUR}_{premiums}) - \text{DUR}_{assets}$$

as the latter is defined at any point in time whereas the former isn’t, because of alternating signs.

A more risk management oriented approach could be

$$Mismatch_{risk} = DUR_{benefits+costs} – (DUR_{premiums} + DUR_{assets}) = DUR_{benefits+costs} – DUR_{premiums+assets}$$

which combines cash-flows of the same sign and avoid undefined values. This approach is also more business related as benefits and costs have to be financed either with premiums or other asset returns if premiums are not sufficient.

There is, of course, one problem with the above formulas, and that is, that the Duration operator is not linear, that means $DUR_{a+b} \neq DUR_a + DUR_b$ in general (if $PV(a) \neq PV(b)$). A general identity is given by:

$$(PV(a)+PV(b))DUR_{a+b} = PV(a)DUR_a + PV(b)DUR_b$$

The summation formula works for positive cash-flow vectors a and b, but there is no general formula for differences of cash-flow streams ("*not sure whether this is really true*") as it is not guaranteed that at any point in time the future part of the cash-flows have always the same sign (in other words, the duration of a-b is not defined if at any point in time t $PV(a(t)) = PV(b(t))$.

That problem does not arise in practice when applied to portfolios as in (almost?) all portfolios the cash-outflows exceed the cash-inflows. such that future cash-flows always stay positive (in a liability sense).