HOW TO MONITOR ACTUARIAL ASSUMPTIONS REGARDING CLAIM FREQUENCY IN PRACTICE?

STÉPHANE LOISEL holds a PhD in applied mathematics from the University of Lyon, an MSc in actuarial science and finance, and is a fellow and former member of the board of the Institut des Actuaires. He is now full professor and head of LSAF research lab at ISFA, Université Lyon 1.

BY **STÉPHANE LOISEL**

In insurance, it is important to continuously monitor actuarial assumptions, and particularly the ones regarding claim frequency. If one does not react quickly enough to a claim frequency increase, then one may face adverse selection. In the case of a claim frequency decrease, competition is at stake. With my colleagues, Nicole El Karoui (Sorbonne Universités) and Yahia Salhi (ISFA, Université Claude Bernard Lyon 1), we have identified the optimal monitoring strategy in a so-called modified Lorden criterion framework: under a false alarm constraint (expressed as a minimum average number of claims before false alarm), the goal is to find the optimal stopping time that minimizes the conditional expectation of the number of claims between the changepoint and the detection time, in the worst case scenario (without any a priori assumption on the time at which the change may occur).

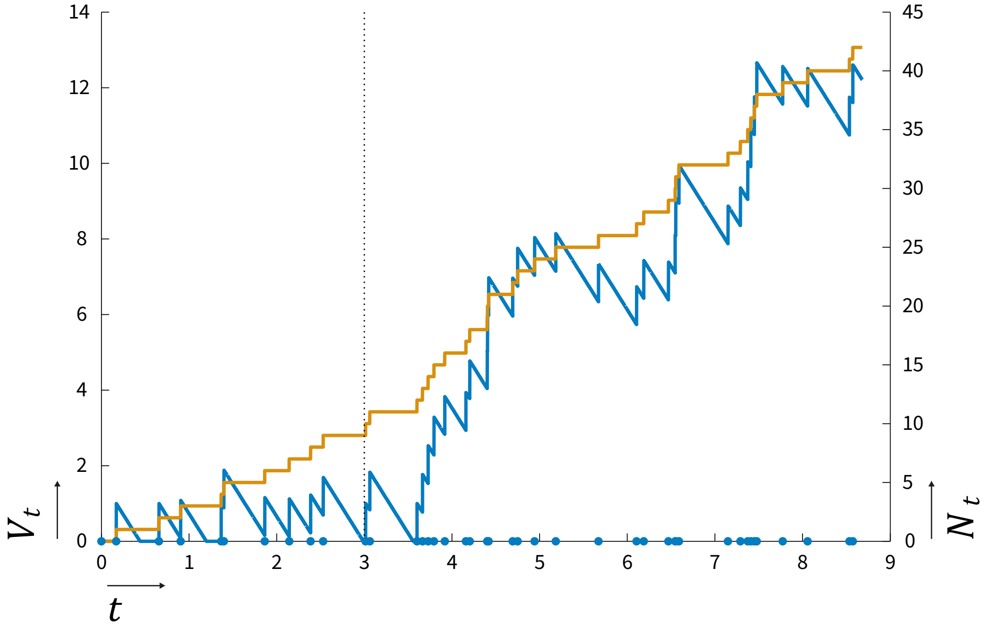

FIGURE 1
Claim arrival process (Nt) (*yellow*) and associated cusum process (Vt) (*blue*) when an increase of claim frequency occurs at time t=3. Before t=3, the cusum process remains close to zero. After the change, there are more upward jumps and the cusum process naturally goes up.

We have proven that the cusum (cumulated sums) process was the optimal tool to monitor claim frequency in this robust optimisation setting. The cusum process corresponds to the logarithm of the Radon-Nydodym density of the probability measure associated to the new model with respect to the one of the initial model. Pathwise speaking, it corresponds to an affine transform of a classical or dual ruin theory process with dividends, where each claim incurs a jump.

When one wants to detect an increase in claim frequency, each claim breeds an upward jump in the cusum process. If claim frequency does increase, then the cusum process is likely to go up (see Figure 1). For a decrease in claim frequency, each claim generates a downward jump in the cusum process. If claim frequency does decrease, then jumps are no longer enough to maintain the cusum process low, and it is likely to increase. In both cases, the alarm is sounded when the cusum process overshoots some upper horizontal barrier, whose level can be determined by the false alarm constraint in closed form thanks to ruin theory computations.

Even if the proof of optimality is a bit technical, the optimal solution is both visual and easy to understand. Besides, the plot of the cusum process can be prolonged as new information comes in. Another merit of the approach is that it provides objective knowledge about optimal opportunities to statistically differentiate a change from pure noise.

Our proof works for a very general class of claim arrival processes, including the ones featuring seasonality, trends (like in mortality patterns for death arrival process monitoring), uncertainty (doubly stochastic Poisson processes for example) or self-exciting properties like in the case of the Hawkes process, very often used to model cyber risk.

Nevertheless, many open research questions remain. What if the change occurs gradually and not in an abrupt manner? What happens if there are multiple or temporary switches from one model to the other? How to handle a portfolio with multiple risks that need to be jointly monitored ?

This research has been supported by the French ANR project LoLitA (Longevity with LifesTyle Adjustments) and the research initiative Sustainable Actuarial Science sponsored by Milliman Paris. To know more :

N. El Karoui, S. Loisel, Y. Salhi, **Minimax Optimality in Robust Detection of a Disorder Time in Poisson Rate**, **Annals of Applied Probability** (2017), Vol. 27(4), 2515-2538.
P. Laub, N. El Karoui, S. Loisel, Y. Salhi, **Quickest detection in practice in presence of seasonality: an illustration with call center data**. In: Data analytics and Models for Insurance, Economica (2020).